Forecasting Modelling by Means of the KPM Method

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The KPM method is a variant of systems dynamics in the Forresterian meaning of the word. It was developed over the years 1973 through 1982 for solving forecasting problems related to the State Programme of Economic Research in Czechoslovakia. In general it may be characterized as an attempt to mesh "quantitative" econometry and "qualitative" systems dynamics. However, the combination of data from time series with expert assessments is not free from problems and has a number of significant methodological implications.

The following text is devoted to the discussion of these problems against the background of several application findings. Starting with a concise characterization of inspiration source of the KPM method, through methodological points of departure, mechanism, description of application possibilities and open problems, it proceeds to characterizing selected application experiences.

INSPIRATION SOURCES

The primary impetus for the formulation and development of the KPM method was given by systems dynamics in the Forres-

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terian sense of the term (Forrester, J.W.: 1961, 1968, 1969, 1971) and by forecasting in the form it was taking in the socialist countries in the early '60s. The specific features of the social system within which the forecasting was taking its shape and developing into an authentic conceptual scheme, marked it with an ineffaceable imprint. Especially significant from our point of view is the fact that macrosocial forecasting research is addressed to the central planning bureau, responsible for the relections and conceptions of long-term strategies and plans for social and economic development of the socialist state. This is why the KPM method protagonists emphasize the implementation functions of forecasting models and participative approach towards their construction.

Obstacles in this connection will be dealt with later in the text. Now, however, let us come back to the inspiration sources of an instrumental nature. These were provided, as already said, by systems dynamics. Its methodological background is well known to the professional public. It was explicitly formulated in individual application works of the above school. The key factors are the following:

The object of modelling is, in most cases, an autonomous social formation with the closed organization (enterprise, town, society, world.)

Its structure is represented as a network of positive and negative feedback loops.

Feedback loops are constituted as a system of mutually interconnected (by material and information bonds levels) and flows. Velocities of the flows are products of the decision-making act.

Levels and velocities of flows, i.e. variables representing thier model expression, are alternating during each run through the system structure.

Level equations have the shape of simple balances. Flow velocity equations are based on the tabulations of partial multipliers, the product of which expresses the so-called total multiplier (synergic effect).

The model as a whole is strictly deterministic. The simulation for the chosen time horizon is carried out step-by-step with varying assumptions and strategies of control. Information generated by the model has a primary social and engineering mission in which it is assisted also by mathematical and programme elementariness of the method.

Compliance with some methodological principles of systems dynamics gives rise to some problems for the practice of modelling, such as:

The object of modelling is not necessarily a *wholesome* autonomous social formation; it may be also a problem sector such as social communication, work, consumption by the population and the like. This may result in the fuzziness of system boundaries and its structure.

Level variables do not necessarily possess a cummulative character (with identifiable increments and decrements); consequently, it is not always possible to define the level in the time t as the level in the time t-1 plus increments minus decrements from t-1 to t.

The state of theories of social objects makes it rarely possible to tabulate partial multipliers on the basis of *a priori* expert information. The synergic effect need not have the form of the product of partial multipliers.

Attempts at a positive solution of the above mentioned problems led, over the last decade, to the autonomy of the KPM method but not to abandoning the class of models within the framework of which it originated and has been implemented. And it must be admitted that these attempts were not invariably successful. It means that the central methodological ideas of systems dynamics are and will be the basic inspiration source of the KPM method.

METHODOLOGICAL BACKGROUND

The product of the application of the KPM method will be designated as the KP model and the process of the construction of the KP model as KP modelling.

The methodological background of KP modelling is characterized by:

basic methodological principles manifested by authors of KP models, the objectification of which, however, need not be complete at all times;

basic methodological points of departure that must be objectified (observed) in order to be still able to speak about KP modelling.

Principles

Priority of theory over the empirical: The form of the initial understanding of social sciences has a decisive influence on the representation of reality through the KP model. If we improve the KP model we are, so to speak, refining model statements concerning the reality within the framework of applied theoretical conception. Consequently, the possibility of devising a KP model on the basis of erroneous theoretical premises cannot be ruled out; we may only believe that verification operations used in KP modelling would, in such a case, result in more signals speaking about the disparity between theoretical and empirical. Situations may arise in which we face aspects of the reality of which we are only beginning to gain a theoretical understanding, or about which there are several competing theories with unequal levels of elaboration and unequal possibilities of mutual complementarity. Alternative strategies of the construction of a KP model with an unknown probability of successful implementation may enter into play here.

The formalization apparatus of a KP model operates with quantitative data on past development trends. Regularities, the identification of which is pursued, are therefore empirical regularities. Theoretical generalizations, if they can be formulated at all, must be made outside the model itself.

Generalization: The model cannot represent particularities, even though they may be important. It may deal only with mass phenomena and, as a result, can be constructed only at a level of generalization that allows for the introduction of the assumption of statistical compensation of fluctuations. We are representing averages rather than particularities.

Dominance of a systems dynamic paradigm: The genetic conditionality by Forrester's models of systems dynamics is reflected also in the dominance of the systems dynamic approach towards the organization of theoretical and empirical information on the object modelled using the KPM method. The principle of dynamics is observed by using data contained in the time series and by the fact that the KP model is able to generate forecasting estimations of future development. The system principle is the basic general-science starting point of the method, although it has a rather general meaning embodying both the postulates of the general systems theory and those of cybernetics, management theory and information theory.

Relative comprehensiveness: The general scientific plane of the basic model paradigm enables the KPM method to be open to knowledge coming simultaneously from various special scientific disciplines. While econometry is defined as a unity of economics, mathematics and statistics, and econometric models are closely related to economics in particular. KP models may be applied wherever the use of one special scientific discipline alone would substantially reduce the interpretation of the reality. It is hardly necessary to stress that efforts aimed at expressing several cognitive aspects through one model face numerous non-trivial cognitive barriers.

Openness of the system: Rather than discouraging us, the awareness of the impossibility of developing a perfect KP model gave us an idea of drawing a correct methodological conclusion from this fact, namely, that we have to provide for the possibility of reformulating the conclusions that have already been adopted as well as the resulting implications over the entire process of the construction and/or application of the KP model. This is done by means of cyclical returns to the preceding stages of KP model creation whenever the next stage is completed, gaining thus additional information on the successfulness of the representation of the social object of the model.

METHODOLOGICAL POINTS OF DEPARTURE

Character of the procedure

If we programmatically give up the necessity of viewing a complex social object only through the prism of one discipline only and prefer a multidisciplinary approach in the interest of a more appropriate model representation, interrelationships underlying the data processed through the model as well as the data themselves become substantially fuzzier, more obscure, "softer". This situation may be dealt with either by adopting "data models" processing quantitative (as a rule statistically obtained) information by means of standard statistical procedures, or by an orientation to "expert models" preferring information heuristically provided by experts on the subject in the form of "tacit knowledge". The general characteristic of KP modelling as a process is an attempt at the synthesis of the two sources of information, their permanent mutual confrontation and control. We consider this synthesis to be feasible because we assume a substitutability, although only limited, of quantitative and qualitative, "data" and "expert" information. In this sense, the KPM method constitutes an attempt at the application of experience obtained both from mathematical and economic models and from models of system dynamics.

Basic input information

From the material point of view, input information may be viewed either as a set of operationalized characteristics (indicators) of the modelled object (the so-called model vocabulary), or as data on the development of such characteristics (time series), and information on assumed interrelationships between these characteristics (model hypotheses). From the information sources point of view we may assume that, at the first delineation, the working variant of the model vocabulary will be provided by the information system of the examined object, which will also enable the necessary data in time series to be obtained. Information of the type of model hypotheses does not, however, stem directly from the information system; rather, it reflects the existing state of knowledge of the object of modelling and, accordingly, has a theoretical or empirical, explicit or implicit form. The explicit form is always assumed in theoretical information. The implicit form is also admitted in empirical information, but it must be gradually made explicit.

A model hypothesis has a primary explanatory function and should account for the development of the characteristic under examination. Consequently it has the character of a statement concerning its assumed determination. If such a statement contains explanatory characteristics that have not been considered in the preceding working variant of model vocabulary, this calls for certain vocabulary modifications even if adequate background data do not exist for the operationalization of these characteristics and their saturation with data. In such a case we must consider substitute means of operationalization, consisting in the use of expert assessments, work with "soft" data, etc. Model vocabulary, in its final version, should represent a set of characteristics making possible the description of the determinant structure of the system as a whole.

From the aspect of definiteness or completeness, we may speak about hypotheses in the continuum of transitions between the so-called minimum shape (requirements for a maximum allowable indefiniteness or incompleteness being given by the character of the model apparatus) and the so-called definition shape with zero degrees of freedom. We must realistically assume that most input hypotheses will have, in their initial form, the character of determinative statements with a relatively low plausibility and definiteness. The KPM method is oriented to this assumption not as a fatal one; rather, it is considered a challenge to seek ways for a gradual increase of plausibility and completeness of hypotheses during the process of model construction.

Fundamental processing of input information

General rules governing the processing of input information in the KPM method are (a) information confrontations, (b) iteration, (c) utilization of the model simulation as one of the instruments for model validation. The common denominator of the above principles is a kind of instrumentalization of methodological scepticism, an attempt to obtain an adequate restructuring of input information during the process of its synthesis, but with an emphasis on creative, heuristic functions.

Confrontation of information may consist, on the one hand, in the confrontation between material information on the same object but coming from different information sources (thus, in the case of confrontation of variant hypotheses we may speak about so-called multi-modelling) and, on the other hand, in the confrontation between different types of material information. The latter is considered as a basic type of confrontation in KP modelling. Emphasis is then laid chiefly on the data (time series) and hypotheses comparison. Data provide information on the manifestations of the system, hypotheses on the factors of such manifestations. Data are thus primarily descriptive, hypotheses primarily explanatory, analytical. In extreme cases the confrontation may result in the rejection of data and hypotheses as incompatible. As a rule, however, the disparity between data and hypotheses provides impulses for restructuring both. The solution of the hypothesis with the use of available data by means of quantitative analysis described in the following pages, consists in the search for a complete (not necessarily right) hypothesis which does not contradict the data (i.e. it is able to reproduce these data) nor does it contradict explicitly theoretical or empirical restricting conditions expressing available knowledge of the character of determination. The conversion of the hypothesis into such complete form (represented by specified functional prescriptions) intensifies eventual disparities between individual information inputs by giving prominence to those limiting conditions that have not been as yet made explicit, and eventually points to inconsistencies or errors in input data measurement.

The iteration consists in the repetition of the solution (whether it is the iterative solution of the hypothesis or of that the previous solution is able, by means of feedback, to restructure with its results the originally used input assumptions. This is the outcome of model conception which, rather than demonstrating, verifies the initial assumptions within the process of the derivation of their consequences and puts substantial requirements on the model apparatus in the direction of an elastic absorption of new explicitly expressed limiting conditions (e.g. conditions expressing the course of the operation of individual factors, but also of the conditions expressing mutual bonds between these factors in their comprehensive operation and, ultimately, of the conditions expressing the specification of factors themselves). Each previous solution constitutes, as a matter of fact, a newly set question, clarified, thanks to the completeness of the output information, but unanswered up to the time when, within the framework of available knowledge, any component of such information may be rejected.

The simulation validation of the model consists in interrelating individual solutions of model hypotheses and transforming them into a set of model equations capable in their totality of simulating the development of the object under examination (real system) and of verifying characteristics of this set during the course of model simulation operations. Isolated solutions of individual hypotheses are intended to acquire and/or intensify their contextual properties during the process of simulation validation. These properties consist partly in the ability to form more complex system formations (such as feedback loops or reproduction cycles), partly in the ability to approximate derived dependencies also outside the reference section given by the previous historical analysis of the development of the system. In this sense, analytical hypotheses are gradually developed into the plane of forecasting hypotheses trying to explain the potentiality of the system under examination, although in a good analytical hypothesis such predictive potential is hidden; the point is, however, to develop systematically the classification and verification of such potential. This verification may have the character of graded simulation tests, starting with the global reproduction of past development, going through simulations of the possible future development with gradually extended horizons (carried to the extreme for heuristic purposes) and ending with a number of variants of anomaly or load tests varifying the impact of external or internal "shocks" on the systemic whole. In other words, we base our considerations on the assumption that the operations of simulation are able to intensify shortcomings of the specification of model equations developed during the preceding stages of model construction. Thus possibility of intensification is seen primarily in such properties of the description of the model as non-linearity, feedback operation, often with multiple transformation and, to a certain extent, also time delays.

Basic output information

Although output information is seemingly generated by the model, it is actually generated by the system "model-user", because the user of the model as bearer of available knowledge is drawn into the play more or less in each step of the iterative solution (in this sense we may speak about dialogic and/or participative modelling). From this point of view, an "invisible" output information is represented also by the cultivation of mental models in both the users and the modelling forecasters.

"Visible" output information may be divided into two mutually interconnected groups. The first group includes information related to the possible future trajectories of the development of the modelled system, having the form of data. The second group includes information related to the possible future determinations of these trajectories having the form of hypotheses. While input information had the form of historical data and primary analytical hypotheses, output information has the form of forecasting data and primary forecasting hypotheses. Output information is updated and revised with any change of internal and external conditions (such as decision-making); new, specific forecasts are developed, conditioned by changing assumptions, and new specific forms of determination (of factor values and their operation) arise within the more general framework of the determination of the systemic entity in its development. Utilization and adjustment of the KP model are permanent in this respect.

FORMALISM AND APPARATUS OF THE KPM METHOD

Typology of definition equations of the KP model

Although the sector of definition equations characterizes, often in the final form, conceptual points of departure of the KP model and its development initiates the work connected with formalization, the specific form of definition equations is simple. The calculation of the basic types of definition equations is therefore given without any further comment and methodological discussion. They include:

- balance equations
- actualization and distribution equations
- interpretation, auxiliary and technical equations.

Balance equations are sufficiently known from the Foresterian models and have the following form:

$$y_t = y_{t-1} + \sum_{i=1}^{n} P_{i,t} - \sum_{j=1}^{m} U_{j,t}$$

where: y – dependent variable
P – variable characterizing the incremental process
U – variable characterizing the decremental process
t – time

Actualization equations (or equation of the actualization of the potential) has the following form:

$$y_t = Q_t \times MA_t$$

where: O – potential
MA – degree of the actualization of the potential

If the observed variable is represented by the product of the distribution of some medium (such as people, money, technology, etc.), O is designated as the basis (source) of the distribution and MA as the distribution coefficient and the equation of the actualization of the potential has been conveniently labelled as "distribution equations".

Interpretation, auxiliary and technical equations cover a relatively varied range of equations – from aggregation equations through equations of exogeneous variables up to correction coefficients or instructions characterizing the initial conditions or assumptions of the given simulation.

Quantitative analysis of hypothetical relationships in the KP modelling

Just like the dynamics or econometry, the KPM method cannot do without an apparatus which makes possible the quantification of model relationships of the type "dependent variable – indepdnent variables".

Forrester's quantification procedure is based on the expert tabulation of influence functions assigned to individual independent variables; the dependent variable is expressed as the product of these functions (the so-called partial multipliers) and homogenization constant.

Using such procedure, *the decisive information sources are the expert's knowledge, experience and intuition.*

Using the regression apparatus, *the decisive information source is constituted by the data obtained through observation* (measurement).

One of the principal ideas of the development of the quantification apparatus for KPM method is a positive unification of the two above procedures. This unification is based on *the thesis of the mutual substitutability of information originating from qualitatively differing sources:* data obtained from observation, knowledge, experience and intuition of the expert.

Basic model — hypothetical equation

It is assumed that the dependency between variables may be expressed by the prescription

$$v(t) = r\left(b + \sum_{i=1}^{n} f_i\,(u_i(t - z_i)) + \epsilon\,(t)\right) \tag{1}$$

where t is time, v is dependent variable, for $i = 1, 2, \ldots, n$ u_i is independent variable (factor) affecting the dependent variable with a lag z_i, f_i is monotonous continuous function (for working purposes called *the sensitivity function*), b is real constant, r is the ascending continuous function (for working purposes called *relation function*), ϵ (t) is random variable with zero mean value and final dispersion. Independent variables are not random quantities.

Algorithm of the quantitative analysis

Estimation of the lag z_i, of sensitivity functions f_i and constant b contained within (1) is obtained through the minimization of the sum of absolute values of deviations $\sum_{v=mz+1}^{m} |d_v|$ in the system given by

the regression nucleus (2)

$$h^{-1}(y_v) = p_y^h + \sum_{i=1}^{n} \sum_{j=1}^{w_i} \sum_{k=z_i^d}^{z_i^h} a_{ijk}\,(g_{ij}(x_{v-k}^{(i)}) - p_{ijk}) + d_v$$

$$p_{ijk} = \frac{1}{m-mz} \sum_{u=mz+1}^{m} g_{ij}(x_{u-k}^{(i)}) \quad p_y^h = \frac{1}{m-mz} \sum_{u=mz+1}^{m} h^{-1}(y_u)$$

$$v = mz + 1, mz + 2, \ldots, m \quad mz = \max\,\{z_i^h\,; i \in N\}$$

$$N = \{1, 2, \ldots, n\}$$

limiting conditions

$$a_{ijk} \geqslant 0 \quad i \in R \quad j = 1, 2, \ldots, w_i \quad k = z_i^d, z_i^d + 1, \ldots, z_i^h \tag{3a}$$

$$a_{ijk} \leqslant 0 \quad i \in S \quad j = 1, 2, \ldots, w_i \quad k = z_i^d, z_i^d + 1, \ldots, z_i^h \tag{3b}$$

$$R \cup S = N \quad R \cap S = \phi$$

$$h^{-1}(v^{\max}) \geqslant p_y^h + \sum_{i=1}^{n} \sum_{j=1}^{w_i} \sum_{k=z_i^d}^{z_i^h} a_{ijk}(g_{ij}(u_i^r) - p_{ijk}) \tag{4a}$$

$$h^{-1}(v^{\min}) \leqslant p_y^h + \sum_{i=1}^{n} \sum_{j=1}^{w_i} \sum_{k=z_i^d}^{z_i^h} a_{ijk}(g_{ij}(u_i^s) - p_{ijk}) \tag{4b}$$

$$0 \leqslant \sum_{j=1}^{w_1} \sum_{k=z_1^d}^{z_1^h} a_{1jk}(g_{1j}(x_r^{(1,k)}) - g_{1j}(x_s^{(1,k)})) + \tag{5}$$

$$+ \sum_{j=1}^{w_2} \sum_{k=z_2^d}^{z_2^h} a_{2jk}(g_{2j}(x_s^{(2,k)}) - g_{2j}(x_r^{(2,k)}))$$

$$0 \leqslant \sum_{j=1}^{w_1} \sum_{k=z_1^d}^{z_1^h} a_{1jk}(g_{1j}(x_r^{(1,k)}) - g_{1j}(x_s^{(1,k)})) +$$

$$+ \sum_{j=1}^{w_3} \sum_{k=z_3^d}^{z_3^h} a_{3jk}(g_{3j}(x_s^{(3,k)}) - g_{3j}(x_r^{(3,k)}))$$

$$\cdot \quad \cdot \quad \cdot$$

$$0 \leqslant \sum_{j=1}^{w_1} \sum_{k=z_1^d}^{z_1^h} a_{1jk}(g_{1j}(x_r^{(1,k)}) - g_{1j}(x_s^{(1,k)})) +$$

$$+ \sum_{j=1}^{w_n} \sum_{k=z_n^d}^{z_n^h} a_{njk}(g_{nj}(x_s^{(n,k)}) - g_{nj}(x_r^{(n,k)}))$$

$$0 \leq \sum_{j=1}^{w_2} \sum_{k=z_2^d}^{z_2^h} a_{2jk}\left(g_{2j}(x_r^{(2,k)}) - g_{2j}(x_s^{(2,k)})\right) +$$

$$+ \sum_{j=1}^{w_3} \sum_{k=z_3^d}^{z_3^h} a_{3jk}\left(g_{3j}(x_s^{(3,k)}) - g_{3j}(x_r^{(3,k)})\right)$$

$$\cdot \quad \cdot \quad \cdot$$

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$$\cdot \quad \cdot \quad \cdot$$

$$0 \leq \sum_{j=1}^{w_{n-1}} \sum_{k=z_{n-1}^d}^{z_{n-1}^h} a_{(n-1)jk}\left(g_{(n-1)j}(x_r^{(n-1,k)}) - g_{(n-1)j}(x_s^{(n-1,k)})\right) +$$

$$+ \sum_{j=1}^{w_n} \sum_{k=z_n^d}^{z_n^h} a_{njk}\left(g_{nj}(x_s^{(n,k)}) - g_{nj}(x_r^{(n,k)})\right)$$

and *the selective condition:* (6)

For any constant $i \in N$, $j = 1, 2, \ldots, w_i$, $k = z_i^d, z_i^d + 1, \ldots, z_i^h$ only one $a_{ijk} \neq 0$ at the most is valid.

For $i \in N$, $\mathcal{X}^{(i)} = [x_1^{(i)}, x_2^{(i)}, \ldots, x_m^{(i)}]^T$ is the series of observations of i factor in equidistant time intervals (steps), similarly $\mathcal{Y} = [y_1, y_2, \ldots, y_m]^T$ is the time series of the dependent variable.

$0 \leq z_i^d \leq z_i^h \leq m$ is the expert-established minimum and maximum number of steps which determine *the set of feasible lags* $Z_i = \{z_i^d, z_i^d + 1, \ldots, z_i^h\}$ for i factor, $mz = \max_{i \in N} \{z_i^h\}$ *is the maximum feasible lag.*

Let $\mathcal{X}^{(i,k)} = [x_{mz-k+1}^{(i)}, x_{mz-k+2}^{(i)}, \ldots, x_{m-k}^{(i)}]^T$, $x_{min}^{(i,k)}$, $x_{max}^{(i,k)}$ express the minimum and maximum components of the vector $\mathcal{X}^{(i,k)}$ for $i \in N$, $k \in Z_i$. The interval $\langle x_{min}^{(i,k)}, x_{max}^{(i,k)} \rangle$ is called *variation interval* of the factor u_i.

$$x_r^{(i,k)} = \left\langle \begin{array}{ll} x_{\max}^{(i,k)} & \text{for } i \in R \\ x_{\min}^{(i,k)} & \text{for } i \in S \end{array} \right. \qquad x_s^{(i,k)} = \left\langle \begin{array}{ll} x_{\min}^{(i,k)} & \text{for } i \in R \\ x_{\max}^{(i,k)} & \text{for } i \in S \end{array} \right.$$

R is the set of indices of those factors for which the expert establishes the non-declining sensitivity functions (assumed increasing influence of the factor), S the set of indices of those factors for which the expert establishes the non-increasing sensitivity functions (assumed decreasing influence of the factor).

Let $x_{\min}^{(i)}$, $x_{\max}^{(i)}$ express minimum and maximum components of the vector $\mathfrak{x}^{(i)}$ for $i \in N$ similarly $y_{\min}$, $y_{\max}$ minimum and maximum components of the vector $\mathfrak{y}$. $u_i^{\min} \leqslant x_{\min}^{(i)}$, $u_i^{\max} \geqslant x_{\max}^{(i)}$ are expert-established *lower and upper feasible bounds of the factor* u_i, $v^{\min} \leqslant y_{\min}$, $v^{\max} \geqslant y_{\max}$ expert-established *lower and upper feasible bounds of the dependent variable.* The interval delimited by lower and upper feasible bounds of the factor u_i /dependent variable/ is called *prognostic interval of the factor* u_i */dependent variable/.*

$$u_i^r = \left\langle \begin{array}{ll} u_i^{\max} & \text{for } i \in R \\ u_i^{\min} & \text{for } i \in S \end{array} \right. \qquad u_i^s = \left\langle \begin{array}{ll} u_i^{\min} & \text{for } i \in R \\ u_i^{\max} & \text{for } i \in S \end{array} \right.$$

h^{-1} is the ascending continuous function on the prognostic interval of the dependent variable. h^{-1} is the function which is inverse to the ascending continuous function $h \in H$, where H is the expert-established final *set of hypotheses of relation functions* which are of relevance for a good estimate of the relation function r.

$g_{ij} \in G_i$, $i \in N$ is the ascending function, continuous on the prognostic interval of the factor u_i from the expert-established set G_i / of the final robustness w_i /of the so-called *problem functions* which are of relevance for a good estimate of the sensitivity function f_i.

The $\hat{f}_i$ estimate of sensitivity function f_i is assumed – see (6) – to have the shape

$$\hat{f}_i\,(u_i(t-k)) = a_{ijk}\,(g_{ij}(u_i(t-k)) - p_{ijk}) \qquad (7)$$

where $j \in \{1, 2, \ldots, w_i\}$, $k \in Z_i$, a_{ijk} are real numbers obtained through the optimization. p_y^h represents the estimate of the constant b.

Optimization task (2) – (6) may be easily converted to the standard task of the selective linear programming (SLP), the (approximate) solution of which may be obtained through the so-called (incomplete) reduction algorithm SLP ([1]).

If the (robustness) $|H| = 1$, we obtain the estimate of all necessary parameters through the optimization of the only system of the type (2) – (6) (hypothesis H is represented by an only function).

If $n_h = |H| > 1$, we form n_h systems of the type (2) – (6), where the representatives of the set H are successively substituted for h. The estimate of necessary parameters is obtained through the selection of one of n_h results using the optimization of systems thus created (e.g. through the selection of the variant with the minimum sum of residua $\sum\limits_{v=mz+1}^{m} |d_v|$).

Let the selected hypothesis of the relation function be h_o, corresponding variable p_y^h be the symbol p_y^o. The following designations are introduced for the resulting variables obtained from the respective system:
Let a_i express the coefficient a_{ijk}, $j \in \{1, 2, \ldots, w_i\}$, $k \in Z_i$ for $i \in N$, such that $a_{ijk} \neq 0$, corresponding variables g_{ij}, k, p_{ijk} are designated as g_i, k_i, p_i, respectively. The estimate of the dependency (1) can be always writen as

$$\hat{v}(t) = h_o\,(p_y^o + \sum_i a_i(g_i(u_i(t-k_i)) - p_i)) \qquad (8a)$$

or

$$\hat{v}(t) = h_o\,(p_y^o + \sum_i \hat{f}_i(u_i(t-k_i))) \qquad (8b)$$

which is called the *resulting hypothetical equation* (RHE).

Construction of the sets of problem functions and of the set of relation function hypotheses

For any $i \in N$ we determine G_i choosing from the previously built *basic set of the functions* Q.

Practical implementation:

The set Q is built on the basis of several distinct types of ascending and continuous functions z on the interval (d, ∞), developed from the functions commonly found in the computer software.

The presently used set Q is built on the basis of thirteen types of z function and contains the total of 174 functions. Functions of the set Q are arranged according to different classification signs, numbered and represented in the *catalogue of graphs.* The set Q is the result of experience obtained in a number of applications of the algorithm of quantitative analysis; with new applications it is improved even further.

Three functions are available for building the set H, namely:

$$\exp(w), \sqrt{2w}, w \ /\text{let } w = b + \sum_{i=1}^{m} f_i(u_i) + \epsilon\ /.$$

Structure of the problem

The *description* of a quantification problem to be solved contains the specification of the dependent variable and factors of the hypothetical equation and the set of data acceptable by the algorithm of quantitative analysis.

This set of data, called *information base*, contains:

Time series of the dependent variable, time series of the factors and the following *complementary information*:

a) Set H of hypotheses of relation functions
b) Sets G_i of problem functions
c) Sets Z_i of feasible lags
d) Direction of factor influence
e) Feasible bounds of the dependent variable and feasible bounds of the factors
f) Sequence of factor influence

To obtain the resulting hypothetical equation, it is formally sufficient to define the optimization problem from the regression nucleus (2) and selection conditions (6).

Subsystems (3), (4), (5) express arbitrary limiting conditions.

By determining the *direction of factor influence*, the expert formulates the subsystem (3).

By determining the *feasible bounds of the dependent variable and factors*, the expert formulates the subsystem (4). Its role is to ensure the inclusion of the set of *functional RHE values* (8) into the prognostic interval of the dependent variable.

By determining the *retrospective sequence of factor influence*, the expert formulates the subsystem of inequalities of the type (5). The sequence of influence corresponds to the sequence of absolute values of differences $\hat{f}_i(x^{(i,k)}_{max}) - \hat{f}_i(x^{(i,k)}_{min})$ in the RHE. Let us note with respect to the notation of the subsystem (5) that it represents the sequence of factor influence corresponding to their order indices.

POSSIBILITIES AND LIMITS OF KP-MODELLING

As in any other methodological scheme that is just beginning to take shape, the KPM method also suffers from a number of serious limitations. We consider it useful to note them as frankly as possible. We shall present them step by step using the methodical procedure of KP-modelling.

The completed applications of the KPM method entailed primarily an attempt at comprehensive and systems modelling and forecasting of objects for which no equivalent or similar qualities existed in theory. It is here that we should seek the rationale of the idea that the model is an instrument capable of filling in blank spots in the world of theory. A pregnant expression of such point of view is the assertion that we are not modelling what we have previously come to know, but we are modelling in order to extend our knowledge. However, in the initial stages of the construction of a KP-model (qualitative analysis of the object of modelling), such an approach forces the modelers to adopt much too pragmatic procedures. The task is formulated as follows: collect the maximum theoretical information on the object of modelling, arrange the information into a synoptical scheme, and adapt this scheme to the requirements of related procedures. In KP-modelling, the above directive (guideline) was implemented primarily through a series of brainstorming sessions and expert assessments according to the previously standardized formulations of the problem.

Operationalization (transformation of theoretical concepts into concepts with empirical meaning) immediately follows the qualitative analysis of the object of modelling. Experts considering the qualitative form of the modelled object, however, only rarely take account of the availability of homogeneous and long range time series, in determining the operational feasibility of the concepts they use. The tension generated between the theoretical and operationalized form of the model is eliminated, in most cases, again by means of expert assessments – oriented on the estimation of absent data. As a result, KP-models are built on data that are unique, but of insufficient validity.

The formulation of model equations (formation of the formalized model) assumes a conceptualized model in the operational form, and collected data. The method used may be designated as the transcription of existing knowledge concerning the object (in the light of operationalization procedure) into the language of formalism used. It is sufficiently known that this is a rigid and in many respects physicalistic language. Especially lacking are the opportunities for an adequate representation of the decision-making subject (endogenous), mechanisms of its functioning, variability of the structures and time lags. An open problem in this class of the models is that of the theoretically justified conception for the identification of synergisms. All these problems largely affect also the form of information obtained in the further steps of the methodical procedure of building a KP-model. However original and flexible may appear, e.g. the algorithm of the quantification of hypothetical relationships in KP-models, the assumption of invariability of the structure, laws of the behaviour, time lags, inadequately developed theory of the synergisms and the like, cause the computer processing of the model produce information too heavily loaded with explicit and implicit assumptions of an instrumental nature. Most frequently implicated are also the much discussed contradiction between the analytical and forecasting functions of the model (is a model that is well descriptive of the history also a good forecasting model? and vice versa), the problem of exogenous determination of the behaviour of modelled system (assumption of the closeness contradicts the character of social objects, while the assumption of the openness increases, by the order of magnitude, the conditionality of forecasts generated by the model, e.g. as a result of the number of exogenous variables, and the quality of accepted forecasts of these variables) and several other problems, the discussion of which accompanies the modelling efforts in social sciences from the very beginning.

It would seem that at this point we might stop listing the problems involved in KP-modelling. The fact that we have at our disposal a functioning model and are aware of its possibilities and limitations is but a prerequisite for a meaningful model creation. This prerequisite can be further developed only in purposefully oriented experimentation on the model – implementation of the model into research or managerial practice. And this is the matter giving rise to most disputes, the poles of which are constituted by the so-called cultivation conceptions of implementation on the one hand and the planning conceptions on the other hand.

Protagonists of the first position view the objective of modelling exclusively as the cultivation of the learning and modelling subject. The learning subject may be naturally represented also by the decision-maker. It is assumed that an educated decision-maker functions within the system of planning and management more efficiently.

Protagonists of the other position put emphasis on the absence of exact methods and procedures in the management and planning of social and economic development. They conceive the KPM method as one of the possibilities for filling this gap. They believe that it is preferable to use an imperfect instrument to using no instrument at all.

Both positions might be questioned. Thus, no decision-maker has been found as yet who would master the mechanism of the functioning of KP-model to such extent that his actual cultivation would take place as the result. There are, however, users who were willing to apply the KP-model in the managerial and planning practice. This, of course, cannot be considered as an argument in favour of protagonists of the purely pragmatic function of KP-models. Some aspects of this problem will be given attention in the next section.

APPLICATIONS OF THE KPM METHOD

Over the ten-year period of the existence of the KPM method, there were a number of application attempts. It would probably be erroneous to classify them as successes or failures. It is more appropriate to speak about completed and uncompleted applications. Causes of the failure of application efforts include primarily the inadequate organizational and material preparation. KP-models were developed, often under the influence of the militant publicity of KP-modelers, by work units lacking the necessary technology, specialists, data banks. Typical are also attempts at using KPM methods before verifying its appropriateness for the solution of the given problem or modelling of the given object.

The mature applications, either with respect to the quality or importance of the object, include especially the KP-models of the Czechoslovak Physical Education and Sports, Area of Labour, Standard of Living. Their brief characteristics are given in the references (Faifr, Vl. – Gál, F. – Potůček, M. – Zeman, M.:1981.) For the sake of completeness we give a few comments on the last of the above mentioned applications.

KP-MODEL OF THE STANDARD OF LIVING*

The first version of the KP-Model of the Standard of Living was developed over the period 1977-1980 (Gál, F. et al.: 1980) and immediately followed also the publication of material and methodological starting points of the improved version (Gál, F. et al.: 1981) as well as the complete description of the KPM method (Faifr, Vl. et al.: 1981). Here, we shall limit ourselves exclusively to the concise characteristic of the material and formal-logical conceptualization of the above model.

The standard of living is, in the improved version of the KP-model, represented as the level of consumption of material goods and services according to individual spheres of needs. Their differentiation is based on the classification used by the Research Institute of the Standard of Living in Bratislava. It

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means that the consumption of material goods and services is observed in: nutrition and eating habits, catering, clothing, housing, health and hygiene, education, transport and communication, culture, recreation and physical education, social assistance. Expenditures (of the population and of the state) connected with the hierarchy according to the scheme in Fig. 1. The formalism used is indicated by the equations 1 through 6. The main equations are those of the actualization of the potential and distribution equations. The degree of the actualization of the given potential, or the degree of the distribution of the given source, occupied in the model structure the position of dependent variables of the equations of hypotheses. The generalized (and thus also simplified) shape of such hypothesis is suggested by notation No. 7. Exogenously, the structure and the trajectories of the development of the standard of living (in the above meaning of the term) were determined by the assumed development of the economic, demographic, scientific and technological potential of Czechoslovakia. These potentials were represented primarily through the generated and used national income, the number and structure of the population and households and the skill level of the workers. Organic components of the relevant environment of the modelled object were represented also by the material and technical conditions of the satisfaction of the above mentioned needs (e.g. the number of flats in the case of housing, and the like).

Scheme No. 1 – Principles (basic) of the construction of the structure of the KP-model of the standard of living

I.	Actualized portion of the potential	=	Potential	×	Coefficient of the actualization of the potential

Example: (Equation 1)

	Market purchases of the population	=	available monetary (cash) incomes	×	Coefficient of the utilization of available monetary incomes for purchases on the market

(Equation 2)
Payments of the population for services = available monetary (cash) incomes X Coefficient of the utilization of available monetary incomes for payments for the services

II. Distribution of the actualized portion of the potential into individual spheres of the needs = Distribution base X Coefficient of the distribution base into individual spheres of the needs

Example: (Equation 3)
Purchases of the population connected with the nutrition = Purchases of the population on the market – total X Coefficient of the distribution of purchases connected with the nutrition in total purchases

(Equation 4)
Purchases of the population connected with clothing = Purchases of the population on the market – total X Coefficient of the distribution of purchases connected with clothing in total purchases

III. Differentiation of any of the need spheres into consumption commodities = Differentiation base X Coefficient of the differentiation of need spheres into commodities

Example: (Equation 5)
Purchases of meat and meat products = Nutrition-related pur-purchases – total X Coefficient of the differentiation of purchases connected with the nutrition to meat and meat products

(Equation 6)
Purchases of milk and milk products = Nutrition-related purchases – total X Coefficient of the differentiation of purchases connected with the nutrition to purchases of milk and milk products

IV.	Conversion of the consumption commodity from the value terms to natural (kind) terms	=	Consumption commodity in value terms	X	Average price

Approximately 75% of dependent variable hypothetical equations were represented by coefficients of the actualization of the potential, coefficients of distribution and coefficients of differentiation. The generalized shape of these equations may be indicated as follows:

(Equation 7)

Coefficient of the actualization of the potential /distribution, differentiation/ = f [Participants of the respective consumer's activity /population total, eventually part of the population/, selected characteristics of these participants /skill/qualification/ level, health condition, etc./, conditions of the participation in certain consumer's activities /offer of products, degree of offered services, etc./, complementary, substitutional and competitive types of expenditures, prices, etc.]

In addition to actualization, distribution and differentiation coefficients, the function of dependent variable hypothetical equations was also fulfilled by the activities associated with the satisfaction of individual needs (such as the attendance at cultural and sports events), natural correlates of selected consumption expenditures (such as the energetic and nutritional value of consumed foodstuffs), etc.

The finalized model was used to carry out a number of experiments of the essentially user's character. Their objective was to study the possible development of the standard of living of the population of CSSR with the given trajectories of the development of economic potential, eventually to identify the disparity between the need for and availability of consumption sources in the CSSR. Results and their interpretation were published and discussed at the central planning authority of the CSSR. But even so we cannot speak about the implementation. The reason is simple – the practice prefers managerial strategies with short-term effects. The long-term perspectives are discussed quite often, but little is done to make something of them.

In the following, we mention some illustrations of the specific forecasting outputs of the KP-model of the standard of living. All we want to do, of course, is to document the functioning of the model. We selected the simulation from 1979 (see Gál, F. et al.: 1980), exogenous assumptions of which are given in Tables 1 and 2. They also include the assumption of the invariability of the development of modelled object. Some of the obtained forecasting information is given in Figures 1 through 7. Their full comprehension, however, should be based on a certain knowledge of the Czechoslovak economic reality, or the reality of the functioning of socialist economics. For example: total expenditures on the satisfaction of needs such as

Table 1
Assumed development of the generated national income and monetary (cash) incomes of the population

Years	Generated national income		Monetary /cash/ incomes of the population	
	% of the growth	mld. Kcs	% of the growth	mld. Kcs
1981 1985	2,7	473 527	2,7	359 399
1986 1990	3,0	543 611	2,8	410 459
1991 1995	3,1	630 711	2,9	472 527
1996 2000	3,2	734 833	3,0	545 613

Table 2
Projection of the development of Czechoslovak population and of the number of census and collective households
(in thousands towards the end of individual years)

Years	Total population	Census households	Collective households
1980	15 382	5 284	5 094
1985	15 475	5 432	5 246
1990	16 058	5 569	5 387
1995	16 416	5 772	5 603
2000	16 866	5 992	5 832

housing, transport and communication or leisure include both the purchases of the population on the market and the payments for services and material social consumption (in varying mutual ratio). In some components, however, one of the above items has a markedly dominant position (for example purchases on the market in the case of nutrition and clothing or material social consumption in the case of health and education).

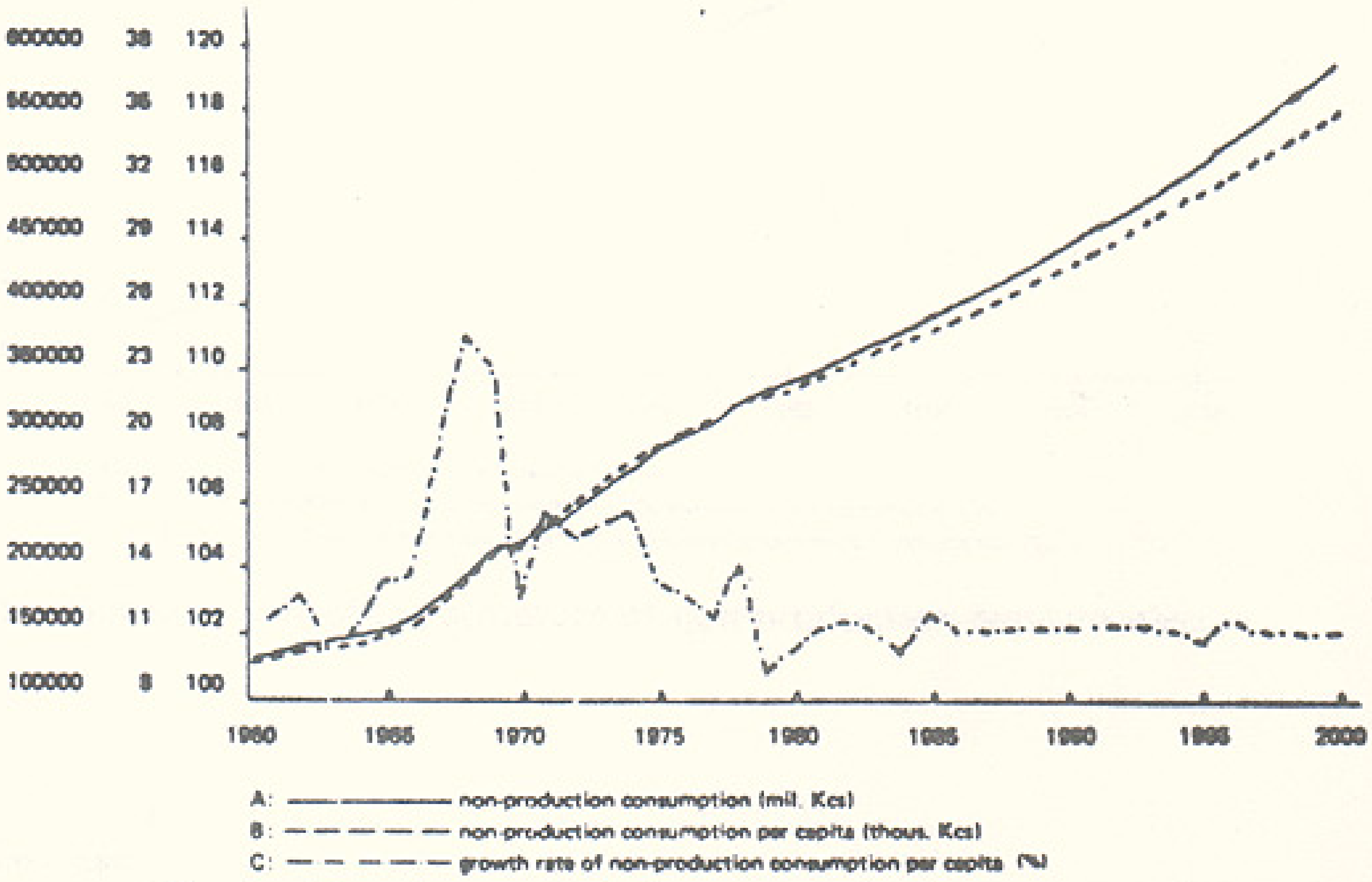

Fig. 1 – Development of the non-production consumption

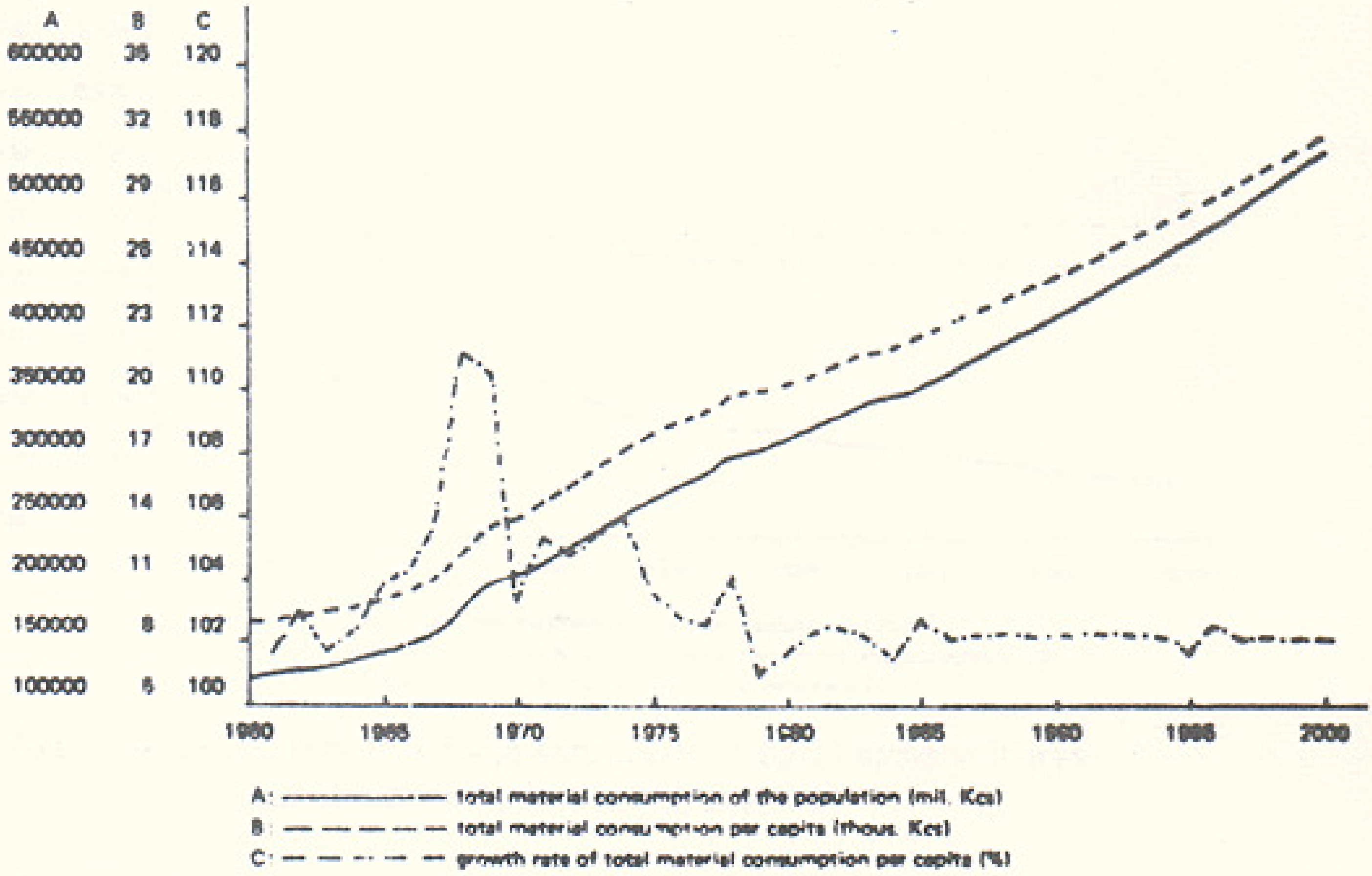

Fig. 2 – Development of the total material consumption of the population

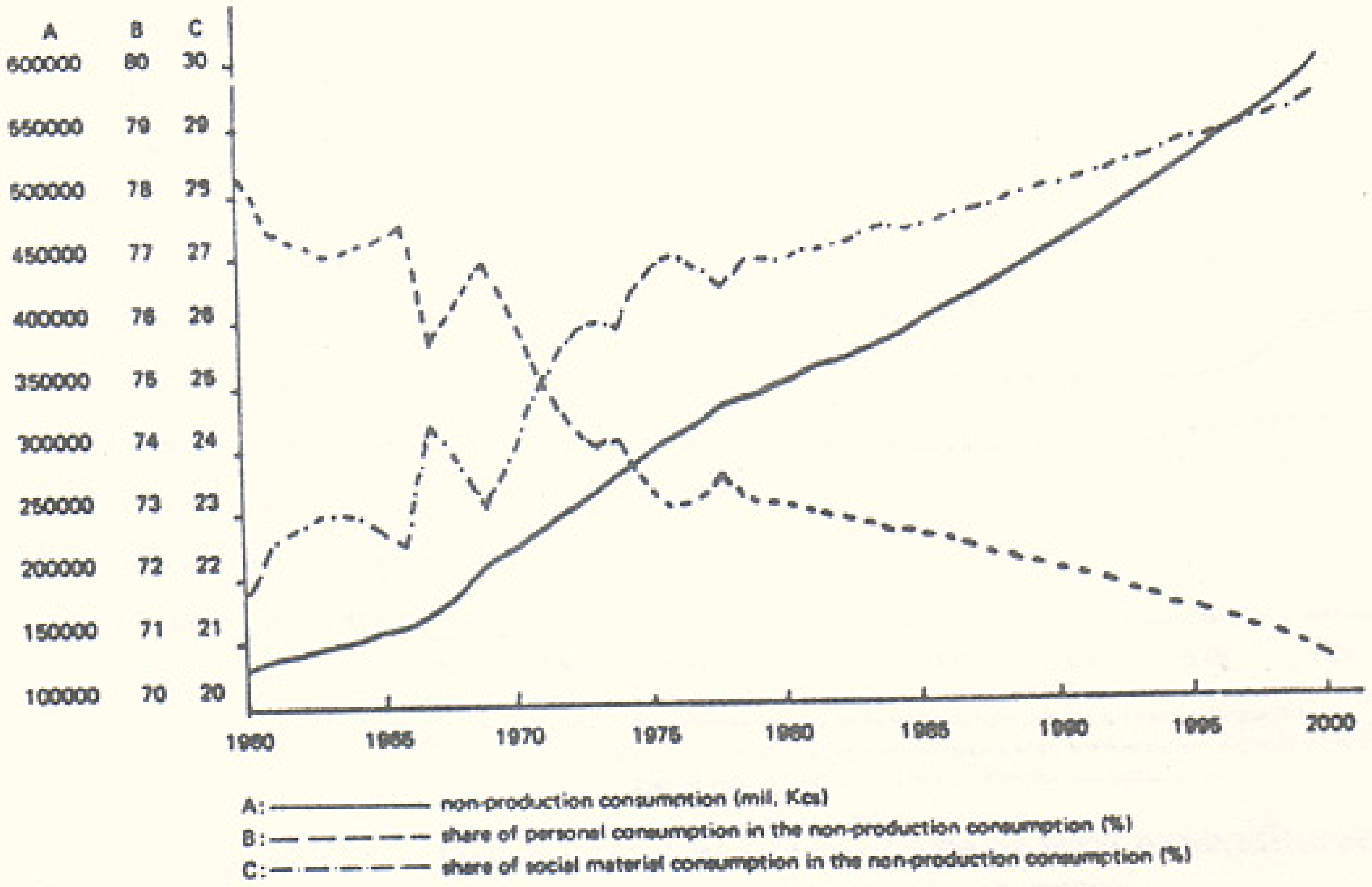

Fig. 3 – Development of the structure of non-production consumption

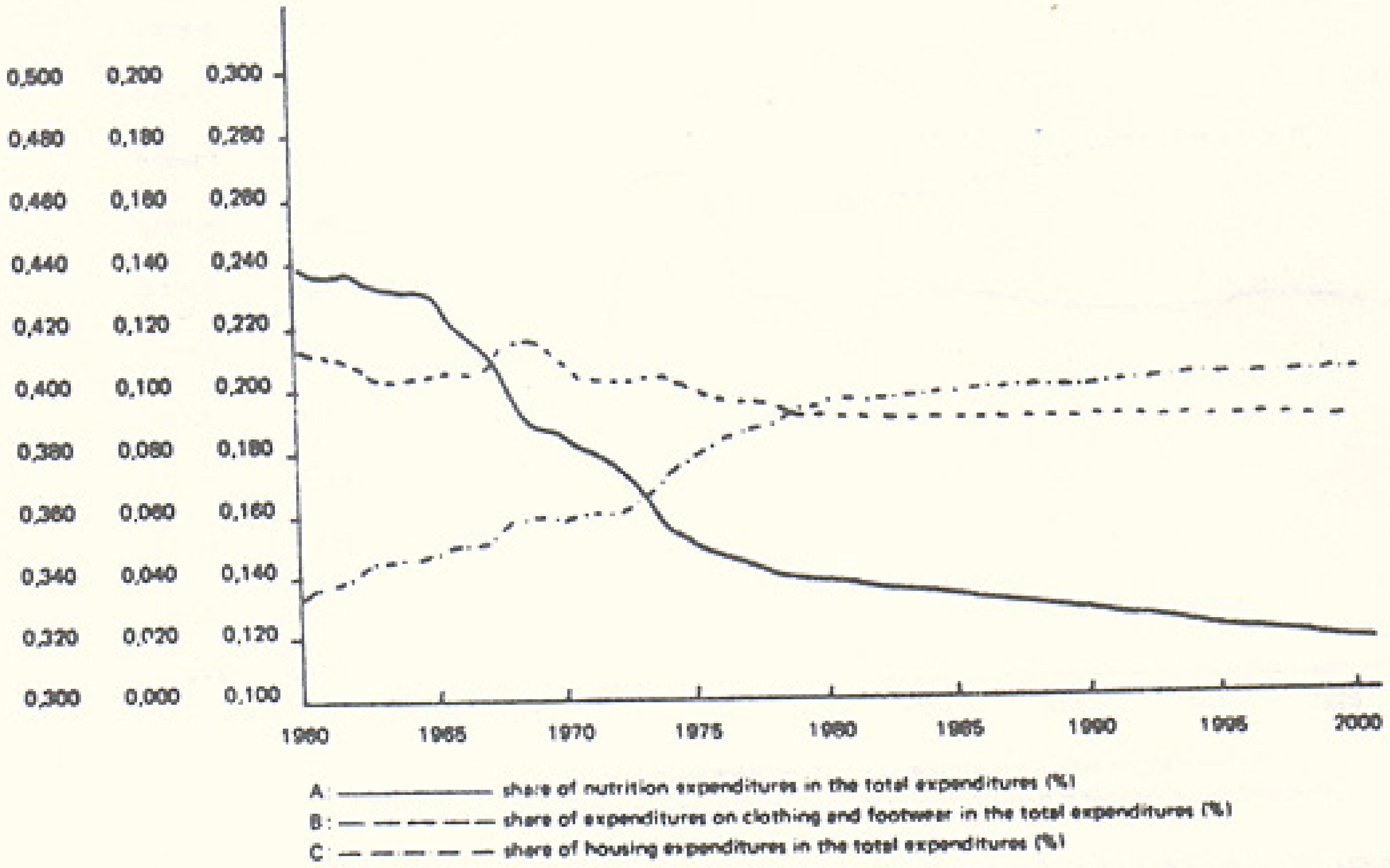

Fig. 4 – Development of the structure of total expenditures (of the population and the state)

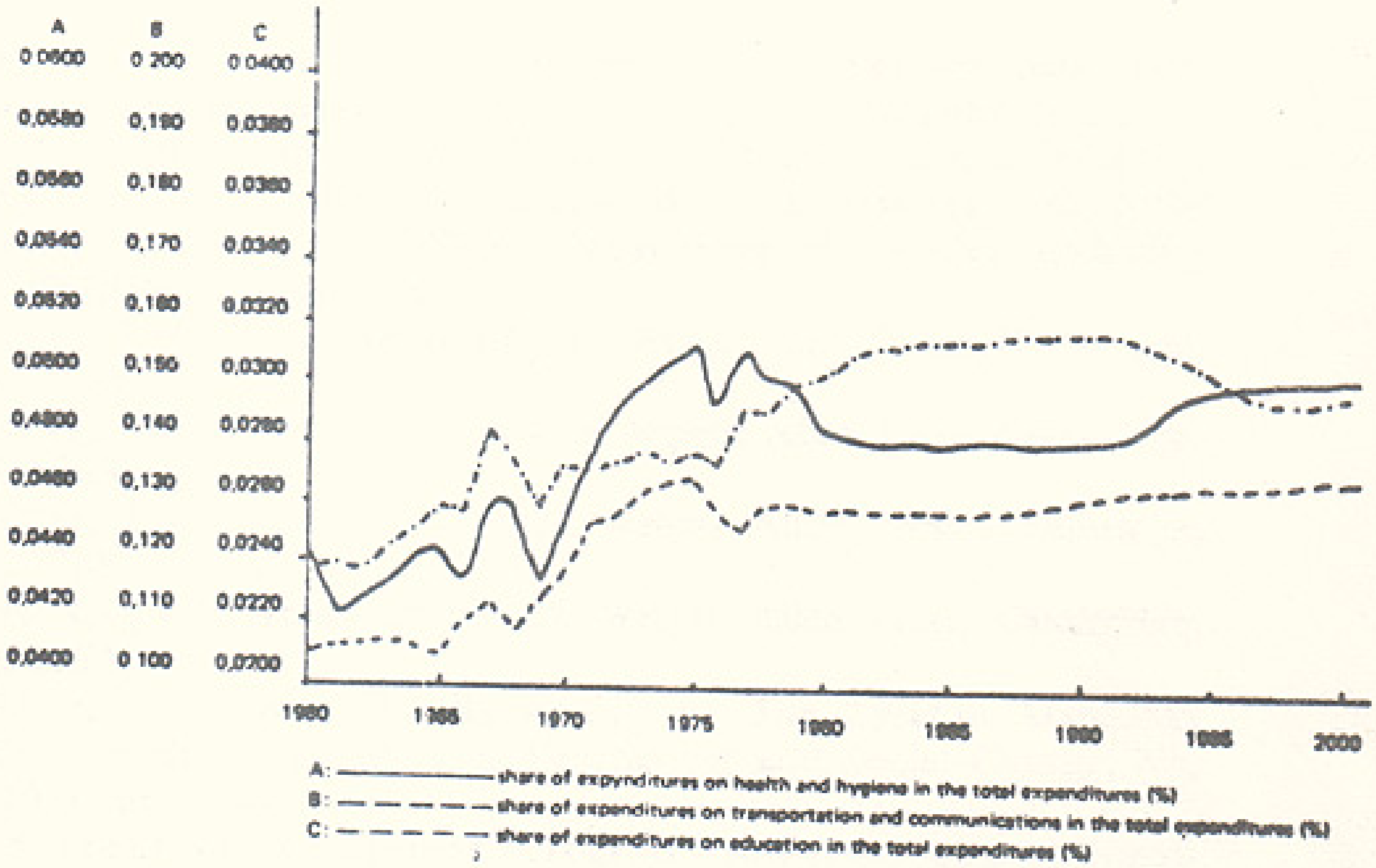

Fig. 5 – Development of the structure of total expenditures (of the population and the state)

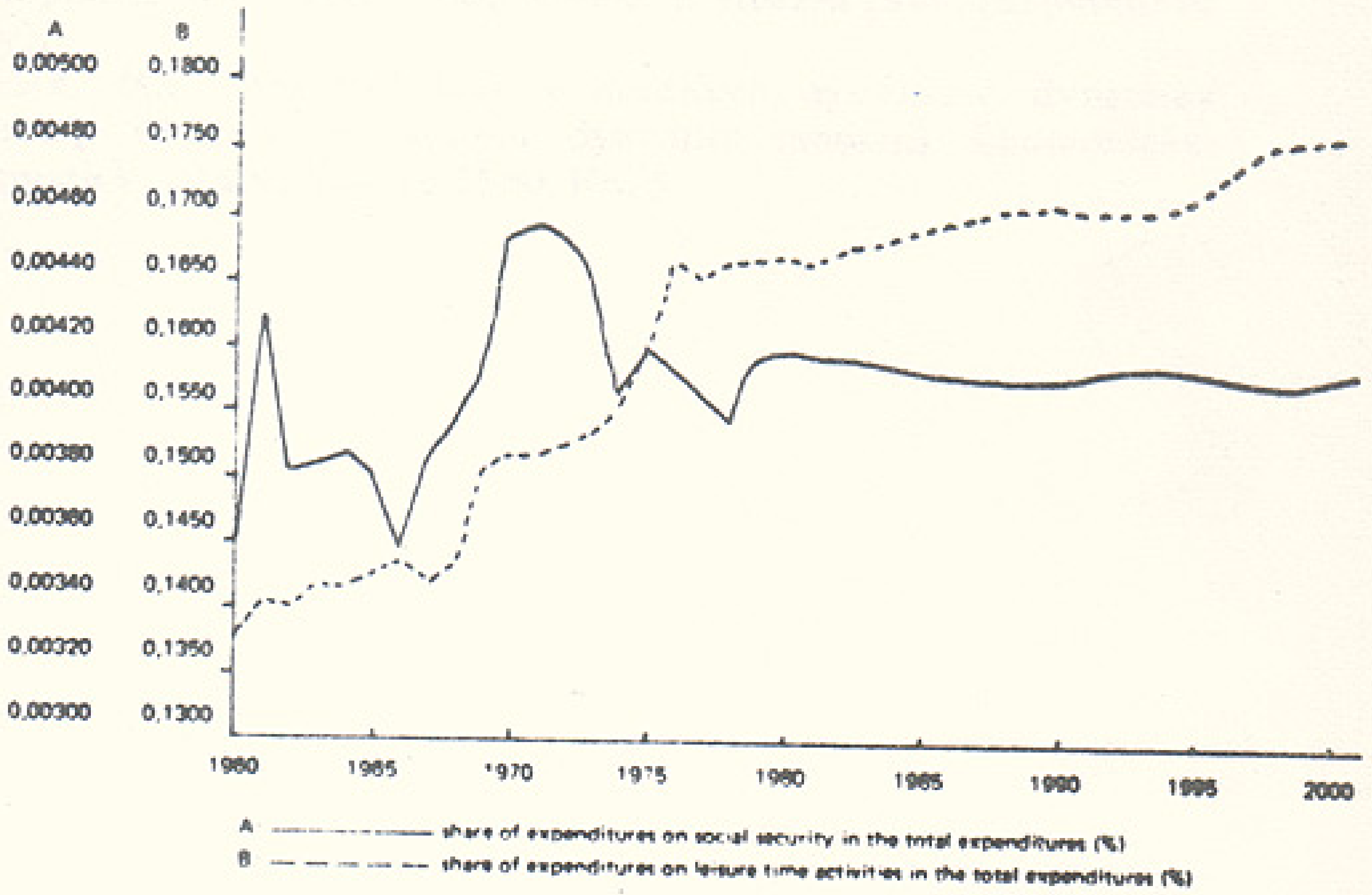

Fig. 6 – Development of the structure of total expenditures (of the population and the state)

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